Abstract

Covers and partial spreads of polar spaces

Andreas Klein and Klaus Metsch

Fachbereich für Mathematik und Informatik Universität Kassel
Heinrich Plett Str. 40 (AVZ)
34132 Kassel

Let $H(3, q^2)$ be a hermitian surface of $\mathrm{PG}(3, q)$. The lines it contains are called its *generators*. An *ovoid* of $H(3, q^2)$ is a set of points of $H(3, q^2)$ meeting every generator exactly once, and a *partial ovoid* is a set of points meeting every generator in at most one point. It is known that $H(3, q^2)$ has ovoids, for example a hermitian curve $H(2, q^2)$ that is obtained by intersecting $H(3, q^2)$ with a non-tangent hyperplane.

It is a simple calculation to see that an ovoid of $H(3, q^2)$ has exactly $q^3 + 1$ points, and it is not difficult to construct a maximal partial ovoid of $H(3, q^2)$ of size $q^3 + 1 - q$. We prove that each maximal partial ovoid has at most $q^3 + 1 - q$ points.

We obtain this result by a careful analyzation of the projective space $\mathrm{PG}(4, q)$ when the blocking sets lives in a degenerate quadric. Counting arguments shows that such a blocking set must have many collinear points unless it is quite large. Our method can be also applied to various other blocking sets or partial spreads.