UNIVERSITÄT BAYREUTH

CA@BT CHAIR OF COMPUTER ALGEBRA

Workshop Program 'Rational Points 2023'

Sunday, July 23

	Arrival
14:00–∼18:00	*Reception is open*
from ∼18:00	*Keys available at the restaurant*
18:30–21:30	*Dinner*

Monday, July 24

08:00–09:15	*Breakfast*
09:20–09:30	Michael Stoll: *Opening and Welcome*
09:30–10:30	Efthymios Sofos: **The second moment method for rational points**
10:30–11:15	*Coffee*
11:15–12:15	Timo Keller: **Anticyclotomic Iwasawa theory and the Birch–Swinnerton-Dyer conjecture for analytic rank 1 at Eisenstein primes of good and bad multiplicative reduction**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Julian Lyczak: **Local solubility of families with multiple fibres**
17:00–17:30	Rosa Winter: **Weak weak approximation for del Pezzo surfaces of degree 2**
17:45–18:15	Victor Flynn: **Arbitrarily Large p-torsion in Tate-Shafarevich Groups of Absolutely Simple Abelian Varieties over $\mathbb{Q}$**
18:30–19:30	*Dinner*

Tuesday, July 25

08:00–09:15	*Breakfast*
09:30–10:30	Stephanie Chan: **The 6-torsion of class groups of quadratic fields**
10:30–11:15	*Coffee*
11:15–12:15	Michael Stoll: **Conjectural asymptotics of prime orders of points on elliptic curves over number fields**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Barinder Banwait: **Towards strong uniformity for isogenies of prime degree**
17:00–17:30	Oana Pădurariu: **The primes of bad reduction of the modular star quotient** $X_0(N)^*$
17:45–18:15	Filip Najman: **Gonality of the modular curve** $X_0(N)$
18:30–19:30	*Dinner*
20:00–?	**Open Problems**

Wednesday, July 26

08:00–09:15	*Breakfast*
09:30–10:30	Natalia García-Fritz: **About the exceptional set in Vojta's conjectures**
10:30–11:15	*Coffee*
11:15–12:15	Héctor Pastén: **Rational curves, p-adic curves, and sparsity of rational points**
12:30–13:30	*Lunch*
	Free Afternoon, *Coffee* and *Dinner* on request

Thursday, July 27

08:00–09:15	*Breakfast*
09:30–10:30	Levent Alpöge: **Conditional algorithmic Mordell**
10:30–11:15	*Coffee*
11:15–12:15	Alexander Betts: **Chabauty–Kim and the Section Conjecture for locally geometric sections**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Alexei Skorobogatov: **Brauer groups of isotrivial varieties**
17:00–17:30	Tony Várilly-Alvarado: **Probabilistic approaches to rational points on algebraic surfaces**
17:45–18:15	Brendan Creutz: **Finite descent obstruction for subvarieties of abelian varieties: weak approximation vs the Hasse principle**
18:30–19:30	*Dinner*
20:00–?	Alex Best and Sander Dahmen: **Lean for the curious arithmetic geometer**

Friday, July 28

08:00–09:15	*Breakfast*
09:30–10:30	Isabel Vogt: **Geometry of curves with abundant low degree points**
10:30–11:15	*Coffee*
11:15–12:15	Nils Bruin: **Arithmetic properties of some low-level quartic modular threefolds**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:30	Steffen Müller: **Algorithms for hyperelliptic Mumford curves**
16:45–17:30	Maleeha Khawaja: **Primitive algebraic points on curves**
17:45–18:15	James Rawson: **Some obstructions to solvable points on higher genus curves**
18:30–19:30	*Dinner*

Saturday, July 29

07:30–09:30	*Breakfast*
	Departure

Abstracts

Efthymios Sofos: **The second moment method for rational points**

In a joint work with Alexei Skorobogatov we used a second-moment approach to prove asymptotics for the average of the von Mangoldt function over the values of a typical integer polynomial. As a consequence, we proved Schinzel's Hypothesis in 100% of the cases. In addition, we proved that a positive proportion of Châtelet equations have a rational point. I will explain subsequent joint work with Tim Browning and Joni Teräväinen [arXiv:2212.10373] that develops the method and establishes asymptotics for averages of an arithmetic function over the values of typical polynomials. Part of the new ideas come from the theory of averages of arithmetic functions in short intervals. One of the applications is that the Hasse principle holds for 100% of Châtelet equations. This agrees with the conjecture of Colliot-Thélène stating that the Brauer–Manin obstruction is the only obstruction to the Hasse principle for rationally connected varieties.

Timo Keller:
Anticyclotomic Iwasawa theory and the Birch–Swinnerton-Dyer conjecture for analytic rank 1 at Eisenstein primes of good and bad multiplicative reduction

We report on work in progress with Mulun Yin. Castella–Gross–Lee–Skinner recently proved Perrin-Riou's Heegner point main conjecture for modular abelian varieties at odd primes p of good reduction for which the mod-p Galois representation ρ_p is reducible ("Eisenstein primes"). They have the restriction that the characters in the semisimplification of ρ_p are non-trivial on $\mathrm{Gal}_{\mathbb{Q}_p}$. For example this excludes the case when there is a non-trivial p-torsion point.

We are working on removing this restriction and generalize the result to newforms of higher weight, allowing us to also treat bad multiplicative reduction using Hida theory. As a consequence, we get the p-part of the Birch–Swinnerton-Dyer conjecture for analytic rank 1 (and 0 with work in progress by Castella–Grossi–Skinner) and a p-converse theorem.

Combining this with previous results of Skinner–Urban, Skinner, Castella–Çiperiani–Skinner–Sprung, we get the strong BSD conjecture in analytic rank 0 and 1 for squarefree level N under a mild condition on the discriminant except maybe for the 2-part.

Julian Lyczak: **Local solubility of families with multiple fibres**

I will report on an ongoing project with Tim Browning and Arne Smeets about the density of everywhere locally soluble members within a family of varieties. In this talk I will focus on the most general case, where components of higher multiplicity are allowed.

Using logarithmic geometry, I will derive exact conditions for a fibre to be locally soluble. Then using the large sieve we obtain upper bounds for the relevant counting problems for several families of examples. In some cases we can also provide lower bounds of the same order, and even asymptotics.

Using these results I will discuss different ways in which the Loughran-Smeets conjecture depends on the absence of multiple fibres.

Rosa Winter:
Weak weak approximation for del Pezzo surfaces of degree 2

Del Pezzo surfaces are classified by their degree d, an integer between 1 and 9. The lower the degree, the more arithmetically complex these surfaces are. It is generally believed that, if a del Pezzo surface has one rational point, then it has many, and that they are well-distributed. After giving an overview of different notions of 'many' rational points and what is known so far for del Pezzo surfaces, I will focus on joint work with Julian Demeio and Sam Streeter where we prove weak weak approximation for del Pezzo surfaces of degree 2 with a general point.

Victor Flynn: **Arbitrarily Large p-torsion in Tate-Shafarevich Groups of Absolutely Simple Abelian Varieties over $\mathbb{Q}$**

We consider Question$_p$: do there exist absolutely simple abelian varieties defined over $\mathbb{Q}$ with arbitrarily large p-part of the Tate-Shafarevich group?

Previously this was only known for $p = 2, 3, 5, 7, 11, 13$. We shall show that the answer is yes for all primes p using an approach for demonstrating arbitrarily large Tate-Shafarevich groups, which only requires the existence of $\mathbb{Q}$-rational p-torsion of rank 2, and does not require an explicit model of any isogenous variety.

(This is joint work with Ari Shnidman.)

Stephanie Chan: **The 6-torsion of class groups of quadratic fields**

For quadratic number fields, the 2-torsion of the narrow class group is well understood by Gauss genus theory. The asymptotics for the average 3-torsion is proved by Davenport–Heilbronn. Using the method of Nair–Tenenbaum, we obtain the correct order of magnitude of the average 6-torsion. This is joint work with Peter Koymans, Carlo Pagano, and Efthymios Sofos.

Michael Stoll:
Conjectural asymptotics of prime orders
of points on elliptic curves over number fields

Define, for a positive integer d, $S(d)$ to be the set of all primes p that occur as the order of a point $P \in E(K)$ on an elliptic curve E defined over a number field K of degree d. We discuss how some plausible conjectures on the sparsity of newforms with certain properties would allow us to deduce a fairly precise result on the asymptotic behavior of $\max S(d)$ as d tends to infinity.

This is joint work with Maarten Derickx.

Barinder Banwait: **Towards strong uniformity for isogenies of prime degree**

Let E be an elliptic curve over a number field k of degree d that admits a k-rational isogeny of prime degree p. We study the question of finding a uniform bound on p that depends only on d, and obtain, under a certain condition on the signature of the isogeny, such a uniform bound by explicitly constructing nonzero integers that p must divide. As a corollary we find a uniform bound on torsion points defined over unramified extensions of the base field, generalising Merel's Uniform Boundedness result for torsion. If time permits we will also demo a command line tool allowing users to generate these uniform bounds for an input d.

This is joint work with Maarten Derickx. [arXiv:2302.08350]

Oana Pădurariu: **The primes of bad reduction of the modular star quotient $X_0(N)^*$**

Let $X_0(N)^*$ be the quotient of the modular curve $X_0(N)$ by the full group of Atkin-Lehner involutions. We know that a prime p is a prime of bad reduction for $X_0(N)$ if and only if p divides N. In joint work with John Voight, we prove that the primes of bad reduction of $X_0(N)^*$ are the same as the primes of bad reduction of $X_0(N)$, outside a finite, explicitly computable set of values for the level N.

Filip Najman: **Gonality of the modular curve $X_0(N)$**

The gonality of a curve C over k is the least degree of a non-constant morphism from C to $\mathbb{P}^1$. In this talk I will present recent work in which we determine the gonality of the modular curves $X_0(N)$ for all $N < 145$ over $\mathbb{Q}$. We also determine all the $X_0(N)$ of gonality 4, 5 and 6 over $\mathbb{Q}$. This is joint work with Petar Orlić.

Natalia García-Fritz: **About the exceptional set in Vojta's conjectures**

Vojta's conjectures with truncated counting functions are far reaching generalizations of the abc conjecture (in the number field case) and the Mason-Stothers theorem (in the function field case). In this talk I will show that in the case of surfaces the exceptional set of Vojta's conjecture for number fields is contained in the exceptional set of Vojta's conjecture for function fields. This allows us to compute the exceptional set in arithmetic problems by using function field arithmetic. This is joint work with Héctor Pastén.

Héctor Pastén: **Rational curves, p-adic curves, and sparsity of rational points**

I will discuss some conjectures on the sparsity of rational points motivated by the work of Manin. These are closely related to the existence of rational curves and the existence of non-constant p-adic analytic curves in varieties. I will give an overview of some existing and new approaches and results.

Levent Alpöge: **Conditional algorithmic Mordell**

In this talk I will specify a Turing machine T and prove the following about it.

1. *On input C/K a smooth projective hyperbolic curve over a number field, if T halts, then its output is $C(K)$.*
2. *The Hodge, Tate, and Fontaine-Mazur conjectures imply T always halts.*

(Joint work with Brian Lawrence.)

Alexander Betts: **Chabauty–Kim and the Section Conjecture for locally geometric sections**

Grothendieck's Section Conjecture predicts that the set of rational points on a smooth projective curve of genus at least two should be equal to a certain "section set", which is defined entirely in terms of the profinite étale fundamental group. Accordingly, one can ask whether the various tools we have for studying rational points on curves can tell us anything about the otherwise mysterious section set. In this talk, I will survey two instances where this idea leads to fruitful mathematics: using the Lawrence–Venkatesh method to understand finiteness properties of the "locally geometric" part of the section set; and using the Chabauty–Kim method to calculate this locally geometric section set in one instance. In the course of the latter, I will explain how we verify infinitely many new cases of a conjecture of Kim.

This is joint work with Jakob Stix, and with Theresa Kumpitsch and Martin Lüdtke.

Alexei Skorobogatov: **Brauer groups of isotrivial varieties**

Consider the generic diagonal surface of degree d, which means that the coefficients are independent variables. We prove that the Brauer group of this surface is equal to the Brauer group of the ground field (in characteristic zero). In contrast, making the coefficients elements of a number field k typically gives a surface with algebraic Brauer group isomorphic, modulo $\mathrm{Br}(k)$, to the cyclic group of order d or $d/2$, depending on the parity of d. This implies, for example, that 100% of all everywhere locally soluble diagonal surfaces of degree d fail weak approximation. Calculations of Brauer groups actually work for more general surfaces in $\mathbb{P}^3$. This is a joint work with Damián Gvirtz-Chen.

Tony Várilly-Alvarado: **Probabilistic approaches to rational points on algebraic surfaces**

The Brauer group of a del Pezzo over a number field is thought to govern the existence of rational points. A large piece of this group is determined by the Galois-module structure on the geometric Picard group of a surface. I will present work in progress that, given equations for a low-degree del Pezzo, determines its algebraic Brauer group with a high degree of confidence. I will also indicate certificates for the probabilistic results. Technology permitting, I will show a live demo.

This is joint work with Austen James.

Brendan Creutz:
**Finite descent obstruction for subvarieties of abelian varieties:
weak approximation vs the Hasse principle**

Stoll conjectured that for smooth proper curves of genus at least 2 over number fields the finite descent obstruction cuts out exactly the set of rational points. He also stated the (a priori weaker) conjecture that such curves have rational points if and only if the set cut out by finite descent is nonempty. In this talk I will explain why these two conjectures are equivalent, and describe similar results that hold more generally for subvarieties of torsors under abelian varieties.

Alex Best and Sander Dahmen: **Lean for the curious arithmetic geometer**

Formalizing mathematics with a computer proof assistant has attracted a lot of attention from research mathematicians in the last few years. We provide an overview of interactive theorem proving, especially focussing on the Lean proof assistant and exciting developments with regards to arithmetic geometry.

For those who cannot get enough of this stuff, we'll stay around to help you get set up and start experimenting yourself with Lean.

Isabel Vogt: **Geometry of curves with abundant low degree points**

An important invariant of a curve defined over a number field is the minimal degree for which it has infinitely many closed points of that degree. In this talk I will discuss joint work with Borys Kadets, extending classification results of Harris–Silverman and Abramovich–Harris, in which we characterize when this invariant takes small values.

Nils Bruin: **Arithmetic properties of some low-level quartic modular threefolds**

Two Siegel modular threefolds of low level allow for birational quartic models in projective space: $A_2(2)$ *is birational to the Igusa (also called Castelnuovo-Richmond) quartic and* $A_2(3)$ *is birational to the Burkhardt quartic.*

In either case, the moduli interpretation can be quite directly constructed geometrically and gives rise to a very concrete representation of the field-of-definition obstruction for genus 2 *curves.*

In the case of $A_2(2)$*, we will also show how the geometry of the quartic can be used to gain some insight in the geometry of* $(2,2)$*-decomposable Jacobians of genus* 4 *curves.*

This talk is based on joint works with Brett Nasserden, Eugene Filatov, and Avinash Kulkarni.

Steffen Müller: **Algorithms for hyperelliptic Mumford curves**

By a theorem of Mumford, a curve defined over a p-adic field with split degenerate reduction can be uniformized using a Schottky group. I will discuss how to make this explicit when the curve is hyperelliptic. Our main application is an algorithm for p-adic heights. This is joint work with Enis Kaya, Marc Masdeu and Marius van der Put.

Maleeha Khawaja: **Primitive algebraic points on curves**

We say a number field K *is* primitive *if* K *and* $\mathbb{Q}$ *are the only subextensions of* K*. Let* C *be a curve defined over* $\mathbb{Q}$ *with genus* g*, with* g *greater than or equal to 2. An algebraic point* P *on* C *is* primitive *if the number field* $\mathbb{Q}(P)$ *is primitive. For instance, quadratic fields are the simplest example of primitive number fields. By a theorem of Hindry and Silverman, if* C *is neither hyperelliptic nor bielliptic then* C *has finitely many quadratic points.*

Let d *be a positive integer. We present some sufficient conditions for* C *to have finitely many primitive points of a fixed degree. On the contrary, we show that if* d *is big enough (with respect to* g*) then* C *has infinitely many primitive degree* d *points, given the existence of one such point.*

This is joint work with Samir Siksek.

James Rawson: **Some obstructions to solvable points on higher genus curves**

It is known that curves of genus at most 4 have points defined over solvable field extensions. I will discuss how to relate such points on curves of genus at least 5 to the Bombieri-Lang conjecture; for a fixed solvable group, the variety parameterising points with this Galois group is of general type. Further, I will explain how certain families of such points would show the existence of a morphism from the curve with solvable Galois group.