EXTENDING RATIONAL DIOPHANTINE QUADRUPLES

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Abstract. A *diophantine m-tuple* is an m-tuple $(a_1, \dots, a_m)$ of distinct nonzero integers such that $a_i a_j + 1$ is a square for all $1 \le i < j \le m$. A *rational diophantine m-tuple* is an m-tuple of distinct nonzero rational numbers with the same property. Fermat found the diophantine quadruple $(1, 3, 8, 120)$. It was a long-standing conjecture that no diophantine quintuples exist; this was proved a few years ago. Euler found the first rational diophantine quintuples. The existence of infinitely many rational diophantine sextuples is known, but it is an open problem whether rational diophantine septuples exist.

We will consider the question how many ways there are to extend a given rational diophantine quadruple to a rational diophantine quintuple. Concretely, we will show that Fermat's quadruple can be extended in exactly one way (by a number already discovered by Euler). The problem can be reduced to that of determining the set of rational points on a certain algebraic curve of genus 2, which in turn can be reduced to finding all points with rational x-coordinate on certain elliptic curves over some quartic number fields. I will explain this in some detail.