Abstract

Weighted $\{\delta v_{\mu+1}, \delta v_{\mu}; k-1, q\}$-minihypers, q a cube prime power

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(joint work with Patrick Govaerts)

The Griesmer bound in coding theory states that if there exists a linear $[n, k, d; q]$ code for given values of k, d and q, then $n \geq \sum_{i=0}^{k-1} \lceil d/q^i \rceil = g_q(k, d)$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

An $\{f, m; k-1, q\}$-*minihyper* is a pair (F, w), where F is a subset of the point set of $\mathrm{PG}(k-1, q)$ and where w is a weight function $w : \mathrm{PG}(k-1, q) \rightarrow \mathbb{N} : x \mapsto w(x)$, satisfying:

(1) $w(x) > 0 \Leftrightarrow x \in F$,

(2) $\sum_{x \in F} w(x) = f$, and

(3) $\min_{H \in \mathcal{H}} \left(\sum_{x \in H} w(x) \right) = m$, where $\mathcal{H}$ denotes the set of hyperplanes of $\mathrm{PG}(k-1, q)$.

Minihypers in projective spaces were introduced to study linear codes meeting the Griesmer bound. The existence of particular weighted minihypers is namely equivalent to the existence of linear codes meeting the Griesmer bound.

But minihypers are also interesting for solving geometrical problems. In particular, the class of weighted $\{\delta v_{\mu+1}, \delta v_{\mu}; k-1, q\}$-minihypers, with $v_s = (q^s - 1)/(q-1)$, is interesting for the study of many geometrical problems.

In the talk of Patrick Govaerts, results on these minihypers for q a square prime power were presented. In this talk, we report on these minihypers for q a cube prime power.

An interesting fact about our results for q a cube prime power is that such minihypers might contain (projected) subgeometries $(\mathrm{PG}(3\mu + 2, \sqrt[3]{q}), w)$, but also projected subgeometries $(\mathrm{PG}(3\mu+2, \sqrt[3]{q}), w) \setminus \Pi_\mu$, where $(\mathrm{PG}(3\mu+2, \sqrt[3]{q}), w) \setminus \Pi_\mu$ denotes a projected subgeometry $(\mathrm{PG}(3\mu + 2, \sqrt[3]{q}), w)$ which originally contained a μ-dimensional space $\Pi_\mu = \mathrm{PG}(\mu, p^3)$, and of which the weight of every point of Π_μ is reduced by one in $(\mathrm{PG}(3\mu + 2, \sqrt[3]{q}), w)$.