New Problems of Graph Reconstruction

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We deal with two new problems of graph theory motivated by applications in information transmission, computational biology and chemistry: 1) reconstruction of an unknown vertex of a given graph from the minimum number of vertices of its metric ball of a given radius, 2) reconstruction of an unknown graph of a given family from metric balls of a given radius of all its vertices. These problems consist of finding the minimum restrictions when such a reconstruction is possible and effective reconstruction algorithms under these minimal restrictions.

Solutions of the first problem for any size of radius are found for the Hamming graph (substitutions errors) and for the Johnson graph (transposition errors) [1], for the poset given by subsequences of sequences (deletions and insertions) [2] and for the poset given by multisets of subpartitions of partitions of an integer [3]. A notion of a single error and of a graph with k single errors are given. It is proved that any graph of degree k is a graph with k single errors. Reconstruction of graphs with some single errors on vertex set S_n [4] is of essential interest for evolution problems in molecular biology.

The second problem consists of finding conditions when metric balls of a given radius r, $r > 1$, centred in all vertices of an unknown graph allow one to reconstruct this graph exactly (or up to isomorphism). There exist examples of graphs of girth $2r+2$ which are not reconstructible by metric balls of radius r. On the other hand, it is proved [5] that any graph of girth at least 7 (distinct from a tree of diameter at most 3) is reconstructible by metric balls of radius $r = 2$. There is a conjecture that, for each $r > 1$, any graph without terminal vertices of girth at least $2r + 3$ is reconstructible by metric balls of radius r. Now this conjecture is also proved for $r = 3, 4, 5$.

References

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