Abstract

Homogeneous and ultrahomogeneous Steiner systems

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Given three integers t, k, v such that $1 \leq t \leq k \leq v$, a *Steiner system* $S(t,k,v)$ is a pair $S = (\mathcal{P}, \mathcal{B})$ where $\mathcal{P}$ is a set of v points and $\mathcal{B}$ is a collection of k-subsets of $\mathcal{P}$ (called blocks), with the property that every t-subset of $\mathcal{P}$ is contained in exactly one block of $\mathcal{B}$. For $t = 2$, this is simply a linear space with lines of size k.

Given a positive integer d, a Steiner system S is said to be $d - homogeneous$ if, whenever the substructures induced by S on two subsets S_1 and S_2 of cardinality at most d are isomorphic, there is at least one automorphism of S mapping S_1 onto S_2. S is called $d - ultrahomogeneous$ if each isomorphism between the substructures induced by S on two subsets of cardinality at most d can be extended into an automorphism of S. S is called $homogeneous$ (resp. $ultrahomogeneous$), whenever S is d-homogeneous (resp. d-ultrahomogeneous) for every positive integer d.

We showed that any 6-homogeneous Steiner system is homogeneous, and that any 7-ultrahomogeneous Steiner system is ultrahomogeneous, by classifying them. We also classified the 5-homogeneous and the 6-ultrahomogeneous Steiner systems. Among the examples found are the Mathieu-Witt systems, some Miquelian inversive planes, and an infinite family linked to affine spaces.