Abstract

Independent sets in finite projective groups

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A subset S of a group G is called **independent** if we have $s \notin \langle S \setminus \{s\} \rangle$ for each $s \in S$.

In [2], useful connections between independent sets of a given group and incidence geometries on which that group acts flag-transitively are described. It is therefore meaningful to investigate independent sets for well-known families of groups. Such investigations were started in [1, 2, 3]. In the last paper the authors prove that an independent set in $PSL(2, p)$ has at most 4 elements for p prime. They also show that the size of a maximal independent set is actually 3 when $p \not\equiv \pm 1 \mod 8$ and $p \not\equiv \pm 1 \mod 10$.

Investigating small primes which are not covered by [3] I found that $PSL(2, 11)$ and $PSL(2, 19)$ have many independent sets of size 4. Also that $PSL(2, 29)$ has no independent set of size 4 and $PSL(2, 31)$ has an independent set of size 4 which is unique up to conjugacy.

This unique independent set of size 4 for $PSL(2, 31)$ gives rise to a rank 4 geometry which has many nice properties. We study this geometry and explain its connection with independent sets.

We also use geometry to study independent sets in $PSL(2, p)$ in general and give some open problems in $PSL(n, q)$ for a true prime power q and $n \geqslant 2$.

References

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[3] J. Whiston and J. Saxl, *On the maximal size of independent generating sets of* $\mathrm{PSL}_2(q)$, J. Algebra **258**(2002), 651–657.